



Research Article



## Training-Based Channel Estimation in MIMO Rician Fading Channels

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### ABSTRACT

In this paper, least squares and linear minimum mean squared error estimators for channel matrix estimation in

Rician fading multi-antenna systems is analyzed. The design of mean squared error minimizing training sequences is investigated in Multiple-Input Multiple-Output Rician fading channels. The problem of Multiple-Input Multiple-Output channel estimation has mostly been treated within the context of minimizing the mean squared error of the channel estimate subject to various constraints, such as an upper bound on the available training energy. A more general framework for the task of training sequence design in Multiple-Input Multiple-Output systems is introduced, which can also treat the minimization of channel estimators. It is shown that the proposed framework can be used to minimize the training energy budget subject to a quality constraint on the mean squared error of the channel estimator, a weighted channel estimation mean squared error and the mean squared error of the equalization error due to the use of an equalizer at the receiver (or an appropriate linear precoder at the transmitter).

**Keywords:** Rician fading channel, Channel Estimation, LS, LMMSE, MIMO.

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### ABSTRACT

In this paper, least squares and linear minimum mean squared error estimators for channel matrix estimation in Rician fading multi-antenna systems is analyzed. The design of mean squared error minimizing training sequences is investigated in Multiple-Input Multiple-Output Rician fading channels. The problem of Multiple-Input Multiple-Output channel estimation has mostly been treated within the context of minimizing the mean squared error of the channel estimate subject to various constraints, such as an upper bound on the available training energy. A more general framework for the task of training sequence design in Multiple-Input Multiple-Output systems is introduced, which can also treat the minimization of channel estimators. It is shown that the proposed framework can be used to minimize the training energy budget subject to a quality constraint on the mean squared error of the channel estimator, a weighted channel estimation mean squared error and the mean squared error of the equalization error due to the use of an equalizer at the receiver (or an appropriate linear precoder at the transmitter).

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### I. INTRODUCTION

An important factor in the performance of multiple antenna systems is the accuracy of the channel state information (CSI) [1]. CSI is primarily used at the receiver side for purposes of coherent or semi coherent detection, but it can be also used at the transmitter side and adaptive modulation. Since in communication systems the maximization of spectral efficiency is an objective of interest, the training duration and energy should be minimized. Most current systems use training signals that are white, both spatially and temporally, which is known to be a good choice according to several criteria [2, 3].

However, in case that some prior knowledge on the channel or noise statistics is available, it is possible to tailor the training signal and to obtain a significantly improved performance. This context aims at providing a unified theoretical framework that can be used to treat the MIMO training optimization problem in various scenarios. Furthermore, it provides a different way of looking at the

aforementioned problem that could be adjusted to a wide variety of estimation-related problems in communication systems.

In this paper, TBCE method is studied in the flat Rician fading MIMO channels. We investigate The least square(LS) and linear minimum mean square error (LMMSE) estimators for channel matrix estimation in Rician fading multi-antenna systems is analyzed, and especially the design of mean square error (MSE) minimizing training sequences.

The rest of this paper is organized as follows. Section II introduces the system model. The training sequence optimization in the Rician fading MIMO channels is investigated in Sections III. Simulation results are presented in Section IV. Finally, Section V presents concluding remarks.

### II. SYSTEM MODEL

Let us consider a MIMO system with  $t$  transmitter and  $r$  receiver antennas. It is assumed the block fading model for flat MIMO channels. Each

transmitted block has N sub-blocks which contain training and data symbols as shown in Fig. 1. The frame structure is the same for all transmitting antennas. Training and data symbols are located in the first and end part of the sub-blocks, respectively. In practice, the channel is estimated using training symbols in the training phase. To estimate the MIMO channel in each sub-block, it is required that  $n_p \geq t$  training signals are transmitted by each transmitter antenna. The  $t \times n_p$  complex received signal matrix can be expressed as

$$Y = HX + n \quad (1)$$

Where  $X$  and  $n$  are the complex  $t$  -vector of transmitted sequences on the  $t$  transmit antennas and  $r$  -vector of additive receiver noise, respectively. In MIMO Rician fading channels with  $K$  as Rice factor, the  $r \times t$  matrix of channel,  $H$ , is defined in the following form:

$$H = \sqrt{\frac{1}{k+1}} \cdot H_{ray} + \sqrt{\frac{k}{k+1}} H_{los} \quad (2)$$

The matrix  $H_{ray}$  explains the Rayleigh component of the channel and the matrix  $H_{los}$  describes the channel mean value or the Line of Sight (LOS) component of the channel. The elements of the matrix  $H_{ray}$  are i.i.d. complex Gaussian random variables with the zero mean and the unit variance.

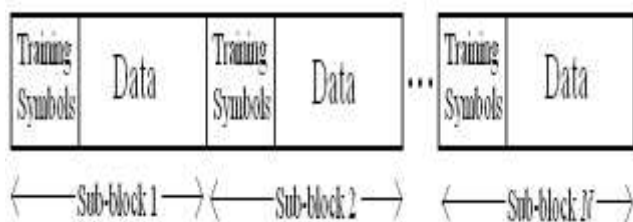


Fig. 1. Frame structure for each transmitting antenna in a MIMO channel.

The MIMO channel model of (1) can be expressed in the following vector form:

$$y = \tilde{X}h + n \quad (3)$$

Where  $y = \text{vec}(Y)$ ,  $z = \text{vec}(n)$ ,  $\tilde{X} = X^T \otimes I_r$  and

$$h = \text{vec}(H) = \sqrt{\frac{1}{k+1}} \cdot H_{ray} + \sqrt{\frac{k}{k+1}} \cdot H_{los} \cdot H \text{ has the}$$

following  $r_t \times r_t$  correlation matrix:

$$R_h = E\{hh^H\} = \frac{1}{1+k} \begin{bmatrix} 1+k & \dots & K \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ K & \dots & 1+k \end{bmatrix} \quad (4)$$

Using equations

$$h = \text{vec}(H) = \sqrt{\frac{1}{1+k}} h_{ray} + \sqrt{\frac{k}{1+k}} h_{los} \text{ and (4),}$$

The  $r_t \times r_t$  co-variance matrix of the Rician fading MIMO channel can be written as

$$C_h = R_h - E\{h\}E\{h\}^H = \frac{1}{1+k} I_{r_t} \quad (5)$$

#### A) ML Channel Estimator

In classical estimation, the channel is assumed to be unknown deterministic. For linear model of the ML estimator which maximizes the joint probability distribution function (pdf) of is optimal [5]. In classical estimation, the channel is assumed to be unknown deterministic. For linear model of the ML estimator which maximizes the joint probability distribution function (pdf) of is optimal.

$$K(y:h) = \frac{1}{\det(C_v) \prod^{mp}} \exp\left[-(y - \tilde{X}h)^H C^{-v} (y - \tilde{X}h)\right] \quad (6)$$

For noise vector in (3), the co-variance matrix is

$$C_v = R_v = E\{vv^H\} = I_{mp} \quad (7)$$

#### B) LMMSE Channel Estimator

For linear model, the MMSE and LMMSE estimators are identical. So, let us obtain a linear estimator that minimizes the estimation MSE of  $H$ . It can be expressed in the following general form:

$$\hat{H}_{LMMSE} = M + (Y - MX) \cdot A_o \quad (8)$$

Where  $A_o$  has to obtained from

$$J_{LMMSE} = E\left\{\left\|H - \hat{H}_{LMMSE}\right\|_F^2\right\} \quad (9)$$

The optimal  $A_o$  can be found from  $\frac{\partial J_{LMMSE}}{\partial A_o} = 0$

and is given by

$$A_o = \left(X^H C_H X + \sigma_n^2 N_R I_{NP}\right)^{-1} X^H C_H \quad (10)$$

The LMMSE estimator of  $H$  can be rewritten in the following form using (9) in (7):

$$\hat{H}_{LMMSE} = M + (Y - MX) \left( X^H C_H X + \sigma_n^2 N_R I_{NP} \right)^{-1} X^H C_H \quad (11)$$

Note that in the Rayleigh fading channel,  $M = 0$ ,  $C_H = R_H$ . The performance of this estimator is measured by the error matrix  $\varepsilon = H - \hat{H}_{LM}$  whose pdf is Gaussian with zero mean and the covariance matrix.

$$C_\varepsilon = R_\varepsilon = E \left\{ \varepsilon^H \varepsilon \right\} = \left( C_H^{-1} + \frac{1}{\sigma_n^2 N_R} X X^H \right)^{-1} \quad (12)$$

Therefore, the error of MMSE estimation can be computed as

$$J_{LMMSE} = E \left\{ \left\| H - \hat{H}_{LMMSE} \right\|_F^2 \right\} = E \left\{ t_r \left( \varepsilon^H \varepsilon \right) \right\} = t_r \left( C_\varepsilon \right) = t_r \left( C_H^{-1} + \frac{1}{\sigma_n^2 N_R} X X^H \right)^{-1} \quad (13)$$

### III. TRAINING SEQUENCE OPTIMIZATION IN MIMO

Let the training matrix  $P$  represent the training sequence. This matrix fulfils the total power constraint  $t_r(P^H P) = p$  which represents the maximal number of spatial channel directions that the training can excite. The combined received matrix of the training transmission is

$$Y = HP + N \quad (14)$$

Where the combined disturbance matrix  $N$  is uncorrelated with the channel  $H$ . Next, we consider the problem of designing the training sequence to optimize the performance of the MMSE estimator in (11). The performance measure is the MSE and the optimization problem can be formulated as

$$\min_P t_r \left\{ \left( R^{-1} + \left( P^T \otimes I \right) \right)^H S^{-1} \left( P^T \otimes I \right)^{-1} \right\} \quad (15)$$

Observe that the MSE depends on the training matrix  $P$  and on the covariance matrices of the channel and disturbance statistics, while it is unaffected by the mean values. Thus, the training matrix can potentially be designed to optimize the performance.

### IV. SIMULATION RESULTS

In this section, our goal is to compare the performance of the LS and LMMSE channel estimators in the spatially correlated Rayleigh and Rician fading channels, numerically. We assume a  $2 \times 2$  flat fading MIMO system, i.e.,  $N_R = N_T = 2$ . The SNR

is defined as  $\frac{p}{\sigma_n^2}$ , as a performance measure, we

consider the channel MSE, normalized by the average channel energy as

$$NMSE = \frac{E \left\{ \left\| H - \hat{H} \right\|_F^2 \right\}}{E \left\{ \left\| H \right\|_F^2 \right\}} \quad (16)$$

The performance of the MMSE estimators and the training sequence design will be illustrated numerically.

In Fig. 1 we give the normalized MSEs averaged over 5,000 scenarios with different coupling matrices with  $N_R = N_T = 2$ . The performance of four different estimators with MSE minimizing training matrices are compared: the MVU/ML channel estimator  $\hat{H} = Y P^H \left( P P^H \right)^{-1}$ , the one-sided linear estimator in [8] that was incorrectly claimed to be the linear MMSE estimator. The MVU/ML estimator is unaware of the channel statistics (i.e., non-Bayesian), and it is clear from Fig. 1 that this leads to poor estimation performance. The two-sided linear estimator also performs poorly under the given premises, but can provide good performance in special cases.

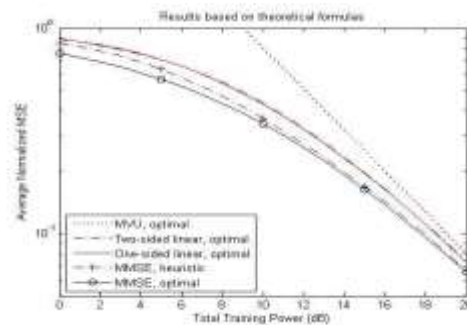


Fig. 2: The average normalized MSEs of channel matrix estimation. The performance of four different estimators with MSE minimizing training matrices is compared.

In Fig. 2, the performance of the MMSE estimator is shown for a uniform training matrix. The MMSE estimator with three different training matrices is compared with the one-sided linear estimator

$$P = \sqrt{\frac{p}{N_T}} I \quad (17)$$

MSE minimizing training matrix (achieved numerically), and the simple explicit training matrix proposed in Heuristic 1. The one-sided linear estimator is given as a reference. We used the coupling matrix to describe an environment with two small scatterers, two big scatterers, and one large cluster. It is clear that the gain of employing an MSE minimizing training sequence is substantial, and the heuristic approach captures most of this gain

although uniform training is asymptotically optimal at high training power.

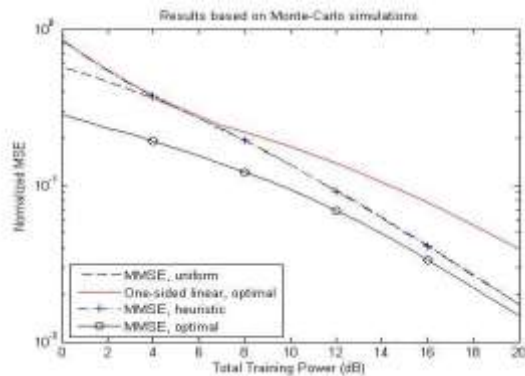


Fig. 3: The normalized MSEs of channel matrix estimation as a function of the total training power in a system. The MMSE estimator with three different training matrices is compared with the one-sided linear estimator.

The average optimal training sequence length (i.e., average rank of) is shown in Fig. 3 for both an MSE minimizing training matrix and the training matrix proposed in Heuristic 1. The average length is given as a function of the total training power and for the spatial correlation induced by  $\alpha \in \{0.33, 0.66, \text{ and } 1\}$ . The optimal training length decreases and the convergence towards full rank become slower. The heuristic training approach is clearly overestimating the training length.

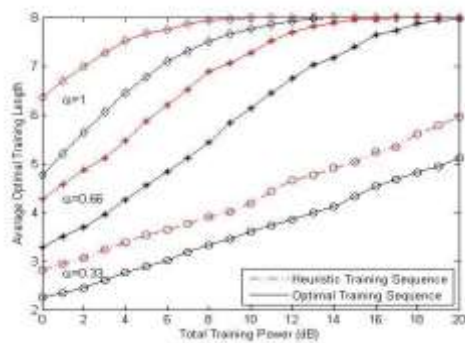


Fig. 4: The average optimal training sequence length (smallest length that minimizes the MSE) as a function of the total training power  $p$ .

## V. CONCLUSIONS

For training-based estimation of Rician fading MIMO channel matrices has been introduced, for the purpose of joint analysis under different noise and interference conditions. The MMSE estimator was analyzed in terms of the MSE minimizing training sequence and the optimal training structure. Finally, the framework was extended to MMSE estimation of the channel, using the same type of training sequences as for channel matrix estimation.

Although the MSE of this estimator can be non-convex, the limiting solutions at high and low training power were derived.

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